

## Simplification of Boolean Expression using Boolean Algebra

Rules:-

1. Compliment =  $(A')' = A$

2. AND =  $A \cdot A = A$

$A \cdot 0 = 0$

$A \cdot 1 = A$

$A \cdot A' = 0$

3. OR =  $A + A = A$

$A + 0 = A$

$A + 1 = 1$

$A + A' = 1$

4. Distributive law =  $A + BC = (A+B)(A+C)$   
 $A \cdot (B+C) = AB + AC$

5. De Morgan's law =  $(A+B)' = A' \cdot B'$   
 $(A \cdot B)' = A' + B'$

6. Commutative law =  $A \cdot B = B \cdot A$   
 $A + B = B + A$

(Commutative law states changing the sequence of variables)

7. Associative law =  $(A \cdot B) \cdot C = A \cdot (B \cdot C)$   
 $(A+B)+C = A+(B+C)$

Examples

(i)  $AB + BC + \bar{B}C = AB + C$   
 $AB + C(B + \bar{B})$   
 $AB + C[1]$   $\therefore B + \bar{B} = 1$   
 $AB + C = AB + C$

(ii)  $A + \bar{A}B + \bar{A}\bar{B}C + \bar{A}\bar{B}\bar{C}D + \bar{A}\bar{B}\bar{C}\bar{D}E$   
 $= A + \bar{A}(B + \bar{B}C + \bar{B}\bar{C}D + \bar{B}\bar{C}\bar{D}E)$   $\therefore A + \bar{A} = 1$   
 $= A + B + \bar{B}C + \bar{B}\bar{C}D + \bar{B}\bar{C}\bar{D}E$   
 $= A + B + \bar{B}(C + \bar{C}D + \bar{C}\bar{D}E)$   
 $= A + B + C + \bar{C}D + \bar{C}\bar{D}E$   
 $= A + B + C + \bar{C}(D + \bar{D}E)$   
 $= A + B + C + D + \bar{D}E$   
 $= A + B + C + D + E$

(iii)  $X = AB + A(B+C) + B(B+C)$   
 $X = AB + AB + AC + BB + BC$  [Open the Brackets]  
 $= AB + AC + B + BC$   $[ B(1+C)$   
 $= AB + AC + B$   $[ B(1)$   
 $= B + AC$

$$(4) \quad (A+B)(A+C) = A+BC$$

$$(A+B)(A+C)$$

$$AA + AC + AB + BC$$

$$A + AC + AB + BC$$

$$A(1+C) + AB + BC$$

$$A(1) + AB + BC$$

$$A + AB + BC$$

$$A(1+B) + BC$$

$$A(1) + BC$$

$$A + BC$$

$$\text{L.H.S.} = \text{R.H.S.}$$

$$(5) \quad \text{Simplify :- } Z = A [B + C(AB + AC)]$$

$$Z = A [B + ABC + ACC]$$

$$Z = A [B + ABC + AC] \quad \because A \cdot A = A$$

$$Z = A [B(1+AC) + AC]$$

$$Z = A [B + AC] \quad 1+A = 1$$

$$Z = AB + AAC$$

$$Z = AB + AC$$

$$Z = \boxed{A(B+C)}$$

Rule - Complement the whole expression  
i) change all OR (i.e. +) to and ( $\cdot$ )  
ii) AND (i.e.  $\cdot$ ) to OR (+)  
iii) Complement each of the individuals